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Solving Transient Eddy Current Problems with Radial Basis Function Method in Frequency Domain

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Abstract
This paper proposed the frequency domain Radial Basis Function (RBF) method for transient eddy problems. It combines the RBF collocation and the Fourier transforms technique. And the RBF method is utilized to solve the frequency model so as to obtain the approximate solution in the frequency domain. While the Fourier transform realizes the time and frequency transformation. On the basis of proper time-frequency parameter settings and the mono-frequency harmonic field solution by the RBF method, the frequency domain solutions are obtained and then transformed into time domain solutions. The numerical simulation of the transient magnetic field in the aluminum flake board showed that the method is feasible and effective. In addition, the method can maintain good precision under large time and space interval because of the RBF's good approach's ability.

Keywords: *Radial basis function, Transient eddy current, Meshless, Frequency domain.*

1. Introduction

The transient eddy current field computation is of great importance for product design and performance analysis. Recently, meshless methods like EFGM (Element Free Galerkin Method), RBF method and RKPM (Reproducing Kernel Particle Method) are gradually applied in the transient eddy current field. S.A.Viana[1] employed the local radial point interpolation method. K.R.Shao introduced the radial function into boundary element[2~3]. Zhang.Y[4] utilizes multi-quadrics collocation method in time domain. Compared to EFGM, the RBF method takes great advantages of approximate function, calculation model and boundary processing. However, time domain method relies greatly on step size. Because the frequency is an important parameter, especially for system frequency response analysis, we proposed frequency domain RBF method.

Firstly, the transient eddy current model and RBF collocation approximation theory were given in section II. Then in section III, the principles of frequency RBF method was put forward. And finally, the frequency RBF simulation of thin aluminum plate transient magnetic field was presented in section IV.

2. Transient eddy current model and RBF collocation approximation theory

2.1 Transient Eddy Current Model

In transient eddy problems, the quasi-static magnetic field model is built. Generally, the initial boundary value form is:

$$\begin{cases} \nabla^2 u(t,\boldsymbol{x}) = \mu\sigma \dfrac{\partial u(t,\boldsymbol{x})}{\partial t} & , \quad \boldsymbol{x} \in \Omega, 0<t<T \\ u(t,\boldsymbol{x}) = \bar{u}(t,\boldsymbol{x}) & , \quad \boldsymbol{x} \in \Gamma_{\mathrm{u}}, 0<t<T \\ \dfrac{\partial u(t,\boldsymbol{x})}{\partial n} = \bar{q}(t,\boldsymbol{x}) & , \quad \boldsymbol{x} \in \Gamma_{\mathrm{q}}, 0<t<T \\ u(0,\boldsymbol{x}) = u_0(\boldsymbol{x}) & , \quad \boldsymbol{x} \in \Omega \cup \Gamma \end{cases}$$

(1)

where u is the magnetic field intensity or a component of magnetic vector potential. Ω is the solution domain, Γ_{u} and Γ_{q} are respectively the first and second-type boundary conditions; T is the time interval.

2.2 RBF Approximation Theory

The RBF is the continuous real valued function of radial vector. The adopted RBF in this paper is multi-quadric (Abbreviated as. MQ) function, its expression in three-dimension is:

$$\phi(\boldsymbol{x}) = \sqrt{(x-c_x)^2 + (y-c_y)^2 + (z-c_z)^2 + \alpha^2} \qquad (2)$$

Where (c_x, c_y, c_z) is the center of the basis function; $\alpha=\beta\|c_i - c_j\|$ is shape parameter. Considering an unknown function $f(x)$ and its discrete data nodes, the RBF interpolation has the following form:[5]

$$f(\boldsymbol{x}_i)=\sum_{j=1}^{N}\lambda_j\phi(\|\boldsymbol{x}_i-\boldsymbol{c}_j\|),\ i=1,2,\ldots,N \quad (3)$$

When the basis function centers are chosen on the interpolation points, the interpolation matrix can be obtained as:

$$\begin{bmatrix}\phi(\|\boldsymbol{x}_1-\boldsymbol{x}_1\|) & \phi(\|\boldsymbol{x}_1-\boldsymbol{x}_2\|) & \cdots & \phi(\|\boldsymbol{x}_1-\boldsymbol{x}_N\|)\\ \phi(\|\boldsymbol{x}_2-\boldsymbol{x}_1\|) & \phi(\|\boldsymbol{x}_2-\boldsymbol{x}_2\|) & \cdots & \phi(\|\boldsymbol{x}_2-\boldsymbol{x}_N\|)\\ \vdots & \vdots & \vdots & \vdots\\ \phi(\|\boldsymbol{x}_N-\boldsymbol{x}_1\|) & \phi(\|\boldsymbol{x}_N-\boldsymbol{x}_2\|) & \cdots & \phi(\|\boldsymbol{x}_N-\boldsymbol{x}_N\|)\end{bmatrix}\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_N\end{bmatrix}=\begin{bmatrix}f(\boldsymbol{x}_1)\\ f(\boldsymbol{x}_2)\\ \vdots\\ f(\boldsymbol{x}_N)\end{bmatrix} \quad (4)$$

Thus the weight coefficients are available:

$$\boldsymbol{\lambda}=[\boldsymbol{\Phi}]^{-1}\boldsymbol{f} \quad (5)$$

Where $[\lambda]=[\lambda_1,\lambda_2,\ldots,\lambda_N]^{T}$, $[f(x_1),f(x_2),\ldots,f(x_N)]^{T}$. Therefore, the function $f(x)$ at any point $\boldsymbol{x}$ is:

$$f(\boldsymbol{x})=\boldsymbol{\Phi}(\boldsymbol{x})\boldsymbol{\lambda}=\boldsymbol{\Phi}(\boldsymbol{x})[\boldsymbol{\Phi}]^{-1}\boldsymbol{f} \quad (6)$$

3. RBF method in frequency domain

In the time modal Eq.(1), function $u(t,\boldsymbol{x})$ consists of time and space variables with first and second order partial derivatives. So in frequency domain, the above model will be transformed at first, then get the frequency domain solution, and finally we can acquire the time domain solution through inverse transformation.

The corresponding frequency domain model is:

$$\begin{cases}\nabla^2 U(\omega,\boldsymbol{x})=j\omega\mu\sigma U(\omega,\boldsymbol{x}),\boldsymbol{x}\in\Omega,-\infty<\omega<\infty\\ U(\omega,\boldsymbol{x})=\bar{u}(\omega,\boldsymbol{x}),\boldsymbol{x}\in\Gamma_{\mathrm{u}},-\infty<\omega<\infty\\ \dfrac{\partial U(\omega,\boldsymbol{x})}{\partial n}=\bar{q}(\omega,\boldsymbol{x}),\boldsymbol{x}\in\Gamma_{\mathrm{q}},-\infty<\omega<\infty\\ \dfrac{1}{2\pi}\int_{-\infty}^{\infty}U(\omega,\boldsymbol{x})\mathrm{d}\omega=u_0(\boldsymbol{x}),\boldsymbol{x}\in\Omega\cup\Gamma\end{cases}$$

(11)

where $U(\omega,\boldsymbol{x})$ is the Fourier transform function of $u(t,\boldsymbol{x})$. The implementation scheme of the frequency domain RBF method is as follows:

3.1 Time and Frequency parameter settings

In Fourier transformations, the truncating and discretization for all related functions as $\bar{q}(t,\boldsymbol{x})$, $u(t,\boldsymbol{x})$, $\bar{u}(\omega,\boldsymbol{x})$ and $U(\omega,\boldsymbol{x})$ are necessary. According to signal theory, the time step should satisfy $\Delta t\le\pi/\omega_c$ where ω_c is maximum frequency. The frequency step should satisfy $\Delta\omega\le 2\pi/T$ where T is the period. The discrete approximate functions are $\bar{q}(n\Delta t,\boldsymbol{x})$, $u(n\Delta t,\boldsymbol{x})$, $\bar{u}(k\Delta\omega,\boldsymbol{x})$ and $U(k\Delta\omega,\boldsymbol{x})$.

3.2 RBF parameter settings

The RBF parameters include center parameter c_i, shape parameter β_i and collocation points. Assume the number of RBFs and collocation points are respectively N and $M=M_1+M_2$ (where M_1 is the collocation points in domain while and M_2 for the boundary).

3.3 Frequency Solution Calculation

(1) mono-frequency solution: Assume the frequency point is $\omega=\omega_0$ ($k=k_0$), solving the corresponding Helmholtz equations in phasor form[6-7]:

$$\begin{cases}\nabla^2 U(k_0\Delta\omega,\boldsymbol{x}_i)=jk_0\Delta\omega\mu\sigma U(k_0\Delta\omega,\boldsymbol{x}_i),\boldsymbol{x}_i\in\Omega\\ U(k_0\Delta\omega,\boldsymbol{x}_b)=\bar{u}(k_0\Delta\omega,\boldsymbol{x}_b),\boldsymbol{x}_b\in\Gamma_{\mathrm{u}}\\ \dfrac{\partial U(k_0\Delta\omega,\boldsymbol{x}_b)}{\partial n}=\bar{q}(k_0\Delta\omega,\boldsymbol{x}_b),\boldsymbol{x}_b\in\Gamma_{\mathrm{q}}\end{cases} \quad (12)$$

The matrix form is:

$$\begin{bmatrix}\boldsymbol{L}[\phi(\boldsymbol{x}_i)]\\ \boldsymbol{B}[\phi(\boldsymbol{x}_b)]\end{bmatrix}\left[\boldsymbol{\lambda}^{(k_0)}\right]=\begin{bmatrix}jk_0\Delta\omega KU(k_0\Delta\omega,\boldsymbol{x}_i)\\ G(k_0\Delta\omega,\boldsymbol{x}_b)\end{bmatrix} \quad (13)$$

where $\boldsymbol{L}$ and $\boldsymbol{B}$ mean domain and boundary operator. Set approximate solution as $U(k_0\Delta\omega,\boldsymbol{x})=\sum_i\lambda_i^{(k_0)}\phi_i(\|\boldsymbol{x}-\boldsymbol{c}_i\|,\alpha_i)$.

In Eq.(12), we have following equations:

$$\begin{cases}\sum_i\lambda_i^{(k_0)}\nabla^2\phi_i(\|\boldsymbol{x}_i-\boldsymbol{c}_i\|,\alpha_i)=jk_0\Delta\omega\mu\sigma\sum_i\lambda_i^{(k_0)}\phi_i(\|\boldsymbol{x}_i-\boldsymbol{c}_i\|,\alpha_i),\boldsymbol{x}_i\in\Omega\\ \sum_i\lambda_i^{(k_0)}\phi_i(\|\boldsymbol{x}_b-\boldsymbol{c}_i\|,\alpha_i)=\bar{u}(k_0\Delta\omega,\boldsymbol{x}_b),\boldsymbol{x}_b\in\Gamma_{\mathrm{u}}\\ \sum_i\lambda_i^{(k_0)}\dfrac{\partial\phi_i(\|\boldsymbol{x}_b-\boldsymbol{c}_i\|,\alpha_i)}{\partial n}=\bar{q}(k_0\Delta\omega,\boldsymbol{x}_b),\boldsymbol{x}_b\in\Gamma_{\mathrm{q}}\end{cases}$$

So getting the coefficient $\boldsymbol{\lambda}^{(k_0)}$.

(2) Frequency sweeping process, with step $\Delta\omega$ in the range $[-\omega_c,\omega_c]$, to find every frequency point solution $U(k_i\Delta\omega,\boldsymbol{x})=\sum_i\lambda_i^{k_i}\phi_i(\boldsymbol{x})=\sum_i\lambda_i^{k_i}\phi_i(\|\boldsymbol{x}-\boldsymbol{c}_i\|,\alpha_i)$ and its corresponding coefficients $\boldsymbol{\lambda}^{(k_i)}$ similarly.

(3) Combining the component frequency solutions into frequency domain solution, we obtain approximate solution $U(\boldsymbol{k}\Delta\omega,\boldsymbol{x})=\sum_i\lambda_i^{(\boldsymbol{k})}\phi_i(\|\boldsymbol{x}-\boldsymbol{c}_i\|,\alpha_i)$.

3.4 Time Domain Solution

Time domain approximate solution $u(t,\boldsymbol{x})$ can be calculated from the frequency solution. With the inverse Fourier transformation, we get the time domain solution as $u(t,\boldsymbol{x})=\dfrac{1}{2\pi}\sum_k U(k\Delta\omega,\boldsymbol{x})e^{jk\Delta\omega t}\Delta\omega$.

4. Simulation of thin aluminum plate

Considering long thin wire wrapped aluminum plate with thickness of d (In the positive x axis direction) in Fig.1.

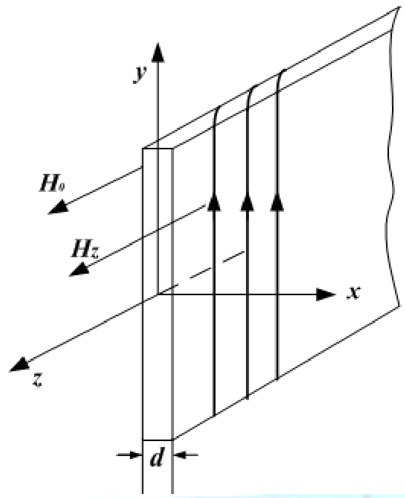

Fig. 1. Model of long straight, thin aluminum plate

The changing process of magnetic field was analyzed for rectangular impulse excitation. The model is:

$$\begin{cases} \dfrac{\partial^2 H_z}{\partial x^2} = \mu\sigma \dfrac{\partial H_z}{\partial t}, & -0.5d < x < 0.5d, 0 < t < T \\ H_z(0,x) = 0, & -0.5d < x < 0.5d \\ H_z(t, \pm 0.5d) = H_0, & 0 \le t \le \tau \\ H_z(t, \pm 0.5d) = 0, & \tau < t \le T \end{cases} \quad (14)$$

The above parameter are d=1.0cm, T=20.0s, τ=2.0s, μ=4π×10^7(H/m),σ=3.82×10^7(S/m) and H_0=1.0A/m. Setting the discrete step $\Delta t = 0.05$s and $\Delta\omega = 0.05 \times 2\pi$ rad/s. The RBF centers and collocation points are equally located in [-0.5cm, 0.5cm] with step h=0. 05cm. Select shape parameter β=5. 0. The test point number is 101 with intervals of 0.01cm. The simulation results are listed in Fig.2 and Fig.3.

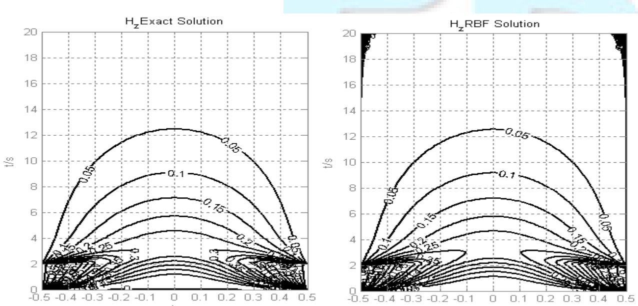

Fig.2 Exact and RBF solutions

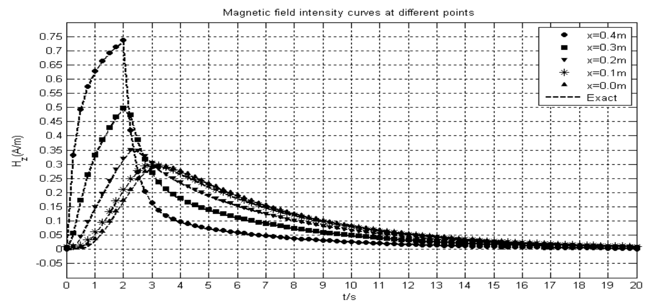

Fig.3 RBF solutions at selected points for magnetic field intensity

To compare errors, define relative RMSE (Root Mean Square error):

$$E_{\mathrm{RMSE}} = \sqrt{\frac{\sum_i (u_i^{\mathrm{exact}} - u_i^{\mathrm{calc}})^2}{\sum_i (u_i^{\mathrm{exact}})^2}} \times 100\%$$

(15)

where u^{exact} is the exact solution, u^{calc} is the approximate solution. The RMSE reflects the proportional relationship of error energy and total energy. In above simulation, the maximum error and the relative RMSE of magnetic field intensity RBF solution are respectively 0.5024A/m and 5.0294%. Tested errors of part of the points selected are listed in table 1:

TABLE 1:Magnetic intensity and its error data at selected points

Point position (cm)	Max_Hz	Max$_{\mathrm{RBF}}$	RMSE$_{\mathrm{RBF}}$
0.45	0.8632	0.0403	1.4505%
0.40	0.7307	0.0126	0.7911%
0.35	0.6103	0.0086	1.0518%
0.30	0.5038	0.0079	1.4766%
0.25	0.4160	0.0081	1.9098%
0.20	0.3499	0.0089	2.2877%
0.15	0.3019	0.0100	2.5784%
0.10	0.2942	0.0107	2.7746%
0.05	0.2917	0.0113	2.8856%
0.00	0.2916	0.0114	2.9211%

Note: Max_Hz is the maximum magnetic field intensity; Max represents the maximum absolute error; RMSE is relative root mean square error.

Besides, after changing truncation length, time interval and step length in simulation, the maximum absolute error and relative RMSE of $H_z(t,x)$ is listed in table 2~3:

TABLE 2:Effect of time interval

Time interval (Δt/s)	Maximum error (A/m)	relative RMSE
0.50	0.5242	20.4480%
0.25	0.5122	12.9752%
0.10	0.5049	7.3466%
0.025	0.5012	3.6523%

TABLE 3:Effect of step length

Step length (*h*/cm)	Maximum error (A/m)	relative RMSE
0.25	0.5024	15.7812%
0.10	0.5024	6.4114%
0.02	0.5024	4.3658%
0.01	0.5024	4.5970%

In the above adjustment of a certain parameter, other parameters than it maintain unchanged (The original simulation parameters were T=20.0s, Δt=0.05s and h=0.05cm). The discontinuity of pulse current along with RBF's limited approximation ability for piecewise functions makes maximum absolute error insensitive with parameter changes. Yet relative RMSE in general reflects the details of the approximate solution. Data we obtained show that: the impact of the truncation length of the error of the most significant, followed by time-distance, and RBF is able to maintain good precision with relatively large step length.

5. Conclusions

This paper introduced a RBF method into transient eddy current field solution; adopted frequency domain processing method for time variables of time-varying field; formed frequency domain RBF method and applied it into thin aluminum plate magnetic field transient analysis. Numerical simulation shows that: frequency domain RBF method is capable and effective for transient eddy current field analysis; moreover, RBF has good approximation ability; thus, it could maintain acceptable precision with relatively larger time interval and step length.

Acknowledgments

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References

[1] Tsuboi, H. Seshima, N. Sebestyen, I. Pavo, J. Gyimothy, S. Gasparics. A, "Transient eddy current analysis of pulsed eddy current testing by finite element method," IEEE Trans. Mag., vol 40, no. 2, pp. 1330- 1333,2004.
[2] Ho, S.L. Shiyou Yang, Guangzheng Ni, Wong, H.C., Yuhuai Wang, "Numerical analysis of thin skin depths of 3-D eddy-current problems using a combination of finite element and meshless methods," IEEE Trans Mag., vol 40, no. 2, pp. 1354- 1357,2004.
[3] Oriano Bottauscio, Mario Chiampi, Alessandra Manzin, "Element-free Galerkin method in eddy current problems with ferromagnetic Media," IEEE Trans Mag., vol 42, no. 5, pp. 1577-1584, 2006.
[4] S.A.Viana, D.Rodger, H.C.Lai. "Application of the local radial point interpolation method to solve eddy-current problems," IEEE Trans Mag, vol 42, no. 4, pp. 91-94,2006.
[5] Yong Zhang, K.R.Shao, Youguang Guo, Jianguo Zhu, D.X.Xie, J.D.lavers. "A comparison of point interpolative boundary meshless method based on PBF and RBF for transient eddy-current analysis," IEEE Trans Mag, vol 43, no. 4, pp. 1497-1500,2007.
[6] Yong Zhang, Shao, K.R. Youguang Guo Lavers, J.D. "A boundary meshless method for transient eddy current problems," IEEE Trans Mag, vol 41, no. 10, pp. 4090-4092,2005.
[7] Zhang, Y. Shao, K.R. Guo, Y. Zhu, J. Lavers, J.D. "Multiquadrics Collocation Method for Transient Eddy Current Problems," IEEE Trans Mag, vol 42, no. 10, pp. 3183- 3185,2006.