Elimination of Harmonics using Resultant Theory

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Abstract

In this paper proposes a multilevel converter with assumed equal dc sources. The multilevel fundamental switching scheme is used to control the needed power electronics switches. Also, a method is presented where switching angles are computed such that a desired fundamental sinusoidal voltage is produced while at the same time certain higher order harmonics are eliminated. Using Fourier Series theory, the transcendental equations eliminating certain higher order harmonics were derived in terms of the switching angles. Furthermore, these transcendental equations were transformed into polynomial equations by making some simple changes of variables. Resultant theory was used to solve the polynomial equations.

Keywords : Harmonics, Fourier Series, Multilevel Converters, Resultant Theory

1 Introduction

In this paper we describe a multilevel converter with assumed equal dc sources is studied. The multilevel fundamental switching scheme is used to control the needed power electronics switches. Also, a method is presented where switching angles are computed such that a desired fundamental sinusoidal voltage is produced while at the same time certain higher order harmonics are eliminated. Using the idea of the Fourier Series, the equations eliminating certain harmonics were derived in terms of the switching angles. In fact, one will see these equations are transcendental equations. By making some simple substitutions, these transcendental equations were transformed into polynomial equations. After forming these polynomial equations, Resultant theory was used to solve the polynomial equations. Furthermore, using the ideas of Symmetric Polynomials and Power Sums, these polynomials were reduced further to form smaller degree polynomials, which are much easier to solve. What makes this approach appealing is that all solutions were found. Numerical techniques, such as Newton-Raphson, will find only one solution. Using the computer algebra software package Mathematica, the aforementioned equations were derived and solved for four different cases. In each case, a different number of dc sources were used with the multilevel converter.

2 Literature Survey

Rioual *et al.* proposed a generalized model in d-q synchronous rotating frames (SRF) to regulate the instantaneous power for PWM rectifier under unbalanced supply voltages. Song and Nam proposed a dual current control scheme in which positive and negative sequence currents are regulated separately by four PI controllers in positive and negative sequence SRFs. In the past decade, most of studies about PWM boost type rectifier under unbalanced operating conditions are based on the control scheme. The rectifier is analyzed in positive and negative d-q SRFs. Reference currents in positive and negative SRFs are obtained by solving a set of non-linear equations. A generalized matrix expression is shown in (1.1).

$$[I_{dq}] = [E_{dq}]^{-1} [S_{in}] \quad (1.1)$$

Suh *et al*. proposed a control scheme to eliminate harmonics in the output DC voltage under generalized unbalanced operating conditions. The proposed technique nullifies the instantaneous ripple power at the rectifier input terminals (BB'). Zero average reactive power is maintained at the supply input terminal (AA') to obtain a unity power factor. Fig. 1 shows the structure and power flow of PWM boost type rectifier.

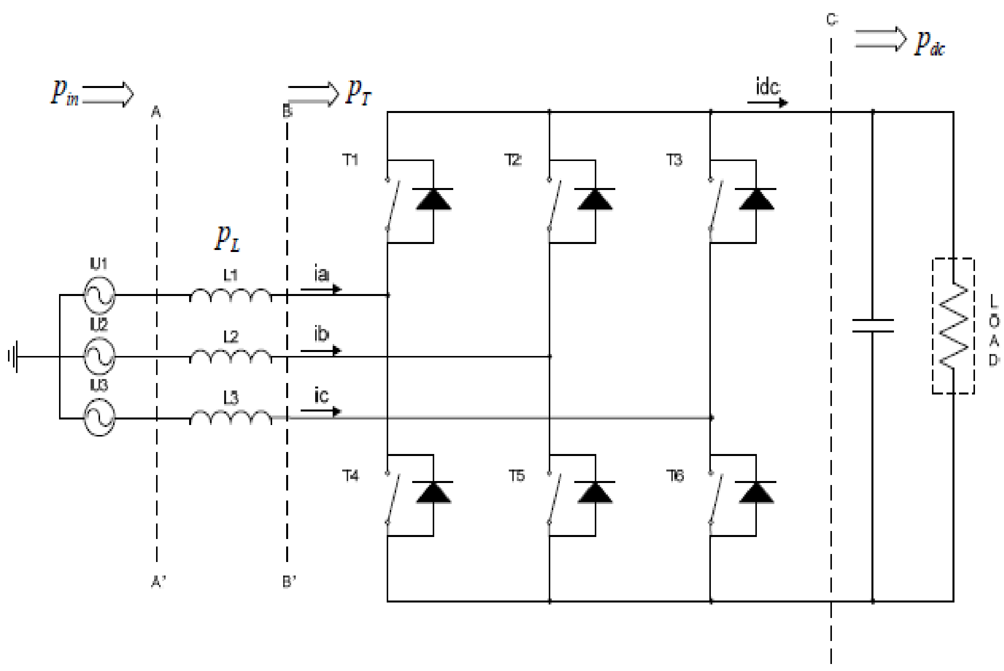

Fig.1 structure and power flow of PWM boost type rectifier.

Yin *et al*. further developed the ideas and proposed an output-power–control strategy which can achieve good performance on both input and output sides of the rectifier. In this method, constant instantaneous power and zero reactive power are both maintained at the rectifier input terminals (BB'). Delivering constant power to DC side ensures a constant DC voltages with no

or negligible low order harmonics. However, the power factor cannot be directly controlled. And there is no result obtained under extremely unbalanced cases.
Wu *et al*. presented a new mathematical model of a three-phase PWM boost type rectifier in the positive and negative SRFs, which can be used to accurately describe the dynamic behavior of PWM rectifier under the balanced and unbalanced operating conditions. However, the control scheme for harmonic elimination and its implementation are still the same as the method in.

3 Multilevel Converters

There are several types of multilevel converters. The three main types of multilevel converters are: diode-clamped multilevel converters, flying-capacitor (also referred to as capacitor-clamped) multilevel converters, and cascaded H-bridges multilevel converters. At this point, it seems appropriate to discuss the difference between the terms "multilevel converter" and "multilevel inverter." The term "multilevel converter" refers to the converter itself. Furthermore, the connotation of the term is that power can flow in one of two directions. Power can flow from the ac side to the dc side of the multilevel converter. This method of operation is called the rectification mode of operation. Power can also flow from the dc side to the ac side of the multilevel converter. This method of operation is called the inverting mode of operation. The term "multilevel inverter" refers to using a multilevel converter in the inverting mode of operation. The main function of a multilevel inverter is to produce a desired ac voltage waveform from several levels of dc voltages. These dc voltages may or may not be equal to one another. The ac voltage produced from these dc voltages approaches a sinusoid. As an example of a multilevel inverter, consider the staircase waveform in Fig. 2

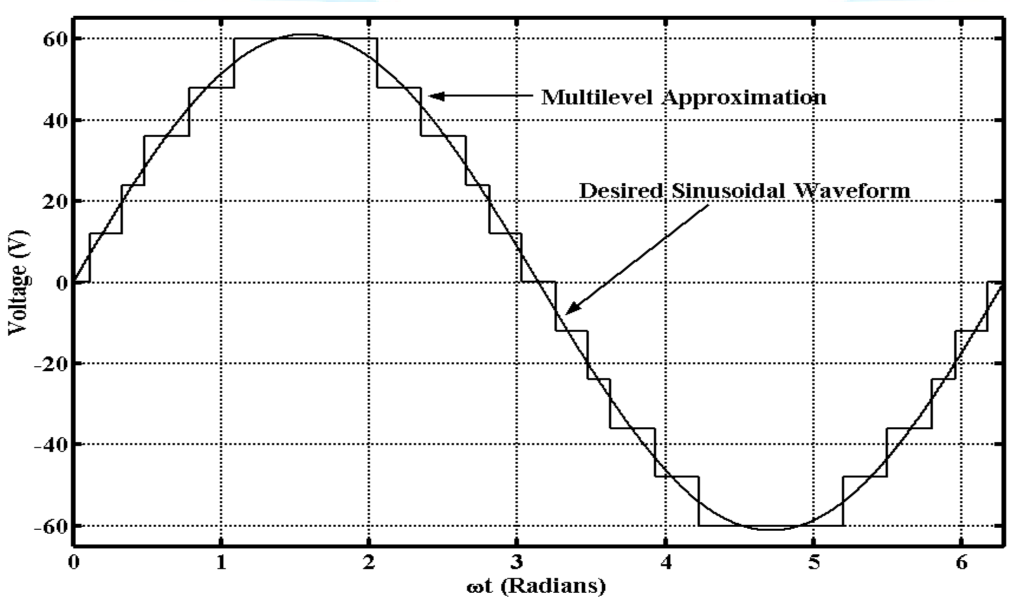

Fig.2 Multilevel inverter using five equal dc sources.

In this figure, five 12 V dc sources produce a staircase waveform with a peak-to-peak voltage of 120 V. In this case, the multilevel inverter produces a fair approximation to a sinusoidal waveform. As one increases the number of dc sources, this approximation will get better and better. Ideally, as the number of dc sources approaches infinity, the staircase waveform will approach the desired sinusoid. Fig.2 also illustrates the "multilevel fundamental switching scheme." This scheme simply refers to determining the switching angles of the multilevel inverter such that a staircase waveform can be produced that approximates a sinusoid. Furthermore, the fundamental frequency of the produced staircase waveform and the frequency of the desired sinusoid are the same.

4 Harmonic Elimination

The multilevel fundamental switching scheme inherently provides the opportunity to eliminate certain higher order harmonics by varying the times at which certain switches are turned on and turned off (i.e. varying the switching angles). Before doing so, one might ask: Why perform harmonic elimination? One reason concerns EMI. Quite simply, harmonics are a source of EMI. As mentioned earlier, EMI can create voltage and current surges. Without harmonic elimination, designed circuits would need more protection in the form of snubbers and EMI filters. As a result, designed circuits would cost more. EMI can also interfere with other "message" signals, such as the control signals used to control power electronics devices. Radio signals are another form of "message" signal that might be affected by the unwanted EMI. Harmonics can also create losses in power equipment. For example, harmonic currents in an electrical induction motor will dissipate power in the motor stator and rotor windings. There will also be additional core losses due to harmonic frequency eddy currents. Harmonics can also lower the power factor of a load. The power factor of a load is proportional to the ratio of the magnitude of the fundamental of the load current to the magnitude of the load current. Increased harmonic content may decrease the magnitude of the fundamental relative to the magnitude of the entire current. As a result, the power factor would decrease. It was mentioned earlier that an increase in the number of dc sources in a multilevel inverter results in a better approximation to a sinusoidal waveform. Furthermore, the increased number of dc sources provides the opportunity to eliminate more harmonic content. Eliminating harmonic content will make it easier to filter the remaining harmonic content. As a result, filters will be easier to design and build. Also, filters will be smaller and cheaper.

5 Fourier Series

Fourier discovered that it is possible to represent periodic functions by an infinite sum of sine and/or cosine functions that are harmonically related. In other words, each trigonometric term in this infinite series has a frequency equal to an integer multiple of the fundamental frequency of the original periodic function. To express these ideas in mathematical form, Fourier showed that a periodic function can be expressed as

$$f(t) = a_v + \sum_{n=1}^{\infty} a_n \cos\left(2\pi n f_o t\right) + b_n \sin\left(2\pi n f_o t\right),$$

where n is the set of natural numbers 1,2,3,…,∞. Ina, a, and are called the Fourier coefficients. These terms are determined from . The term is the fundamental frequency of the periodic function . The integer multiples of , such as and 3, are known as the harmonic frequencies of.

6 Resultant Theory

The purpose of this section is to discuss Resultant theory. When the multilevel fundamental switching scheme is implemented using s switching angles, it can be used to derive s different harmonic equations. In other words, s switching angles will be used to control the values of s different harmonics. Unfortunately, these harmonic equations are transcendental equations, making them difficult to solve without making use of some sort of numerical iterative technique, such as Newton-Raphson. However, by making some simple changes of variables and simplifying, these transcendental equations can be transformed into a set of polynomial equations. Then, Resultant theory can be utilized to find all solutions to the harmonic equations.

$$R(x_1)=\det\left(\begin{bmatrix} b_0(x_1) & a_0(x_1) & 0 & 0 & 0 & 0 \\ b_1(x_1) & a_1(x_1) & b_0(x_1) & a_0(x_1) & 0 & 0 \\ b_2(x_1) & a_2(x_1) & b_1(x_1) & a_1(x_1) & b_0(x_1) & a_0(x_1) \\ b_3(x_1) & a_3(x_1) & b_2(x_1) & a_2(x_1) & b_1(x_1) & a_1(x_1) \\ 0 & 0 & b_3(x_1) & a_3(x_1) & b_2(x_1) & a_2(x_1) \\ 0 & 0 & 0 & 0 & b_3(x_1) & a_3(x_1) \end{bmatrix}\right)=0.$$

R $(X_{1)}$ is called the Resultant Polynomial

7 Application of Resultant Theory to Multilevel Inverter

In this section, an example application of Resultant theory will be given by considering a cascaded H-bridges multilevel inverter using three equal dc sources. In this example, the value of the output voltage fundamental will be controlled while the fifth and seventh order harmonics are eliminated.

7.1 Transcendental Harmonic Equations

The following equation gives the values of the odd sine harmonics corresponding to the multilevel fundamental switching scheme using five switching angles. If three switching angles are used instead, it can be shown that the corresponding equation is

$$b_n=\frac{4}{\pi n}V_{dc}\left[\cos(n\theta_1)+\cos(n\theta_2)+\cos(n\theta_3)\right].$$

If one wants to control the peak value of the output voltage to be V and eliminate the fifth and seventh order harmonics, the resulting harmonic equations are:

$$\frac{4}{\pi}V_{dc}\left[\cos(\theta_1)+\cos(\theta_2)+\cos(\theta_3)\right]=V_1,$$

$$\cos(5\theta_1)+\cos(5\theta_2)+\cos(5\theta_3)=0,$$

A harmonic is a triplen harmonic if its frequency is an integer multiple of three times the fundamental frequency. For balanced three-phase systems, each phase voltage will contain triplen harmonics equal in both magnitude and phase to the triplen harmonics of the other two phases. Therefore, all triplen harmonics will cancel in the line-to-line voltages. For this reason, when multilevel fundamental switching is employed, the selected harmonics to be eliminated are usually not triplen harmonics.

8 Solutions to Polynomials Using Resultant Theory

The polynomials $p_1, p_5,$ and p_7 are functions of the variables $x_1, x_2,$

Using p_1 to solve for x_1 in terms of the other two variables, one gets

$$x_1 = m - x_2 - x_3.$$

Substituting this result into the other two polynomials, one gets

$$p_5(x_2,x_3)=\left(5(m-x_2-x_3)-20(m-x_2-x_3)^3+16(m-x_2-x_3)^5\right)$$
$$+\left(5x_2-20x_2^{\,3}+16x_2^{\,5}\right)+\left(5x_3-20x_3^{\,3}+16x_3^{\,5}\right)$$

and

$$p_7(x_2,x_3)=\left(-7(m-x_2-x_3)+56(m-x_2-x_3)^3-112(m-x_2-x_3)^5\right.$$

$$\left.+64(m-x_2-x_3)^7\right)+\left(-7x_2+56x_2^3-112x_2^5+64x_2^7\right)$$
$$+\left(-7x_3+56x_3^3-112x_3^5+64x_3^7\right).$$

After X_1 has been trivially eliminated, one can now apply Resultant theory to eliminate X_2 . For the research presented in this paper, all Resultant calculations were found by using the Resultant command in the software package Mathematica. After factoring and then eliminating redundant factors and unnecessary numerical constants, the Resultant of the two polynomials and was found to be

$$res(x_3) = (6125\,m^2 - 49000\,m^4 + 137200\,m^6 - 179200\,m^8 + 116480\,m^{10}$$

$$-35840\,m^{12} + 4096\,m^{14}) + (-18375\,m + 269500\,m^3 - 1019200\,m^5$$

$$+1691200\,m^7 - 1361920\,m^9 + 501760\,m^{11} - 65536\,m^{13})x_3$$

$$+(12250 - 588000\,m^2 + 3234000\,m^4 - 7156800\,m^6 + 7293440\,m^8$$

$$-3261440\,m^{10} + 491520\,m^{12})x_3^2 + (637000\,m - 5782000\,m^3$$

$$+17875200\,m^5 - 23385600\,m^7 + 12902400\,m^9$$

$$-2293760\,m^{11})x_3^3 + (-269500 + 6370000\,m^2 - 28694400\,m^4$$

$$+49324800\,m^6 - 34298880\,m^8 + 7454720\,m^{10})x_3^4$$

$$+(-4410000\,m + 30184000\,m^3 - 71500800\,m^5 + 63974400\,m^7$$

$$-17776640\,m^9)x_3^5 + (1470000 - 20776000\,m^2 + 72441600\,m^4$$

$$-84940800\,m^6 + 31539200\,m^8)x_3^6 + (9800000\,m - 50176000\,m^3$$

$$+80281600\,m^5 - 40857600\,m^7)x_3^7 + (-2744000 + 21952000\,m^2$$

$$-53939200\,m^4 + 36556800\,m^6)x_3^8 + (-6272000\,m$$

$$+25088000\,m^3 - 20070400\,m^5)x_3^9 + (1568000 - 6272000\,m^2$$

$$+5017600\,m^4)x_3^{10}.$$

Since the polynomial res is only a function of one variable, one can begin the process of finding the appropriate switching angles.
One should notice above that complex roots to the polynomial equations are being considered as candidates for switching angles. The reason is due to the fact that the imaginary part of the root may be small enough such that the real part of the root may still lead to a viable switching angle. Equation gives an indication of the harmonic distortion due to the fifth and seventh order harmonics. Theoretically, should always be zero since one is supposed to be eliminating the fifth and seventh order harmonics. However, it was just mentioned that complex roots might be considered where the imaginary part is infinitesimally small. Nevertheless, these complex roots will lead to a small but nonzero harmonic distortion. Also, numerical round off in the computation of the roots will lead to a small harmonic distortion. The values given can be controlled such that they are always below some arbitrarily small number ε. For the research presented in this paper, this tolerance level was set at 0.001 times the current value of m.

Two more important points should be made. In general, the algorithm for finding the desired switching angles can theoretically be applied to the more general case of s switching angles. It should also be pointed out that the above algorithm can be applied to the Unified Approach switching scheme. However, the process is more complicated. For a multilevel inverter using s dc sources, recall that the Unified Approach switching scheme considers: Unipolar Programmed PWM with s + 1 switching angles, Virtual Stage PWM with s + 1 switching angles, and multilevel fundamental switching with s switching angles. Therefore, two algorithms need to be implemented. The first algorithm finds the solutions corresponding to multilevel fundamental switching with s switching angles. The second algorithm finds the solutions corresponding to Unipolar Programmed PWM and Virtual Stage PWM with s + 1 switching angles.

9 Summary

In this paper, several topics were discussed. The idea of the Fourier Series was first discussed. It was then illustrated how Fourier Series theory could be used to derive the transcendental harmonic equations corresponding to the multilevel fundamental switching scheme. Furthermore, these harmonic equations were written in terms of the switching angles of the multilevel inverter. Resultant theory was then introduced. After the aforementioned transcendental harmonic equations were transformed into polynomial equations, it was shown how Resultant theory could be used to solve these polynomial equations. Finally, an example application of Resultant theory was given where a cascaded H-bridges multilevel inverter using three equal dc sources was considered. In this example, the value of the output voltage fundamental was controlled while the fifth and seventh order harmonics were eliminated. For the special case of when the multilevel inverter is using equal dc sources, it will be shown how the idea of Symmetric Polynomials can be utilized to transform the set of polynomial equations above into a new set of polynomials of lower degree. As a result, it will be easier to solve these equations. Power Sums theory provides another way of transforming the set of polynomial equations above into a new set of polynomials of lower degree. Compared to typical PWM switching schemes, multilevel fundamental switching will lead to lower switching losses. As a result, using the multilevel fundamental switching scheme will lead to increased efficiency. One drawback of using the multilevel fundamental switching scheme is that the created harmonics occur at frequencies around the fundamental. However, appropriate switching angles can be determined such that some of these harmonics are eliminated. As a result, smaller filters can be used to eliminate the remaining harmonics.

9.1 Future Research

One suggestion for future research would be to extend the multilevel fundamental switching scheme to include more than six equal dc sources. For example, perhaps the ideas presented in this thesis could be used to determine the switching angles for a cascaded H-bridges multilevel inverter utilizing seven equal dc sources. In this case, the seventh switching angle would be used to eliminate the 19^{th} order harmonic. However, as mentioned above, increasing the number of switching angles will lead to polynomial equations of higher degree. Even if the ideas of Symmetric Polynomials and Power Sums are used to simplify these equations, they could become too complex to solve.

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